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ABSTRACT

This paper is based on efforts to bring change to school mathematics by trying to develop mathematics classroom communities in predominantly minority classrooms. In these communities, students work towards doing mathematics by working on open-ended, investigative situations; sharing ideas and strategies; and jointly negotiating meanings. Students also need to develop mathematics from their backgrounds and experiences with everyday mathematics. This paper explores the tensions and compromises resulting from the different conceptions of program participants (school and university teacher-researchers, students, and parents) of what mathematics is and of what mathematics children should learn. The work discussed focuses on geometry in a fifth-grade class. An appendix contains written work by students on finding angles on pattern blocks. Contains 38 references. (Author/MKR)

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**Everyday Mathematics, "Mathematicians' Mathematics,"
and School Mathematics:
Can we (Should we) Bring These Three Cultures Together?**

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Abstract

This paper is based on some of our efforts to bring in change to school mathematics by trying to develop mathematics classroom communities in predominantly minority classrooms. In these communities we work towards having children doing mathematics ("like mathematicians") by working on open ended, investigative situations, sharing ideas and strategies, and jointly negotiating meanings (Cobb, 1991; Lampert, 1986; Schoenfeld, 1991). Yet, we also want these communities to develop from the students' backgrounds and their experiences with everyday mathematics (uncovered, for example, via household visits) in an effort to bridge the gap between outside and inside school experiences (Bishop, 1994; Lave, 1988; Nunes, 1992; Saxe, 1991).

This paper explores the tensions and compromises resulting from the different conceptions of what mathematics is and of what mathematics children should learn, that those of us in the project (school and university teachers-researchers, students, parents) have. We do not want their everyday mathematics to serve "simply" as a source of motivation. Yet, how far can we push everyday mathematics? Once we start mathematizing these everyday situations we may be losing what made them appealing in the first place, but we are gaining (hopefully) in advancing the students' learning of generalization and abstraction in mathematics. This work has made us reflect on our different beliefs, values, and practices in mathematics that largely dictate our actions in the classroom.

The paper will largely focus on our work in geometry in a fifth grade class. Geometry allowed us to combine quite successfully elements of a mathematics community with the students' everyday knowledge. It also allowed us to partially tear down the wall that by fifth grade most students have build isolating what counts as school mathematics.

This paper is based on some of our efforts to bring in change to school mathematics by trying to develop a mathematics classroom community in a fifth grade class. At the center of our vision of such a community are the children doing mathematics "like mathematicians." In Schoenfeld's (1987) terms, we would like to "create a microcosm of mathematical culture" (p. 213) in the classroom. The children will work on open ended problems and investigations; they will share ideas and strategies with each other and jointly negotiate meanings; different approaches to problems will be encouraged and valued (Cobb, 1991; Lampert, 1986; Schoenfeld, 1991; Wilcox, Schram, Lappan, & Lanier, 1991). Yet, we also want these communities to develop from the students' backgrounds and their experiences with everyday mathematics (uncovered, for example, via household visits and children's interviews) in an effort to bridge the gap between outside and inside school experiences (Bishop, 1994; Lave, 1988; Nunes, 1992; Saxe, 1991).

Can we combine "everyday mathematics" and "mathematicians' mathematics" in yet a third arena, namely that of "school mathematics"? Are these three different kinds of mathematics? In what ways are they different? In the first part of this paper I highlight what I view as characteristics of these three kinds of mathematics. This provides the framework on which the research presented here is based. I then move to a description and analysis of our work in the fifth grade class. The description is interspersed with a discussion of key points in relation to the question of whether we can combine these three kinds of mathematics in the classroom. In the last section I summarize those points and focus on issues that we faced as we tried to change mathematics learning by bringing together aspects of these "different communities."

But first of all, I will illustrate with a brief example the kinds of issues I am struggling with as we try to bring change to mathematics classrooms along the lines of developing a mathematics learning community that reflects some of the "mathematicians' mathematics" while building on the children's backgrounds and experiences. In our earlier work (Civil, 1992) we used the topic of Money to develop a learning module that would allow children to contribute their knowledge on this theme. The two teachers with whom I collaborated in this module predicted a wealth of knowledge in their students on issues such as bartering, buying and selling, budgeting. The issue was then how to build on this knowledge to help them learn mathematics. We thought, for example, about presenting the students with problem situations around the concept of conversion and rate of exchange (something with which most children in that school were familiar with due to their ties to Mexico¹). We often found ourselves going back and forth between contextualizing the learning of mathematics on these students' everyday experiences and going into other aspects of mathematics--maybe more "abstract"--that we thought these students should

¹ Our work takes place in schools where most of the students are of Mexican origin.

be learning. I guess our plight was (and still is in our current work) how far can we push everyday mathematics to allow students to uncover other aspects of mathematics? In a "replication" of our work with the theme of money a year later, we posed the following problem to fifth graders:

I gave \$1 to the cashier and he gave me back 3 coins change. How much might I have spent?

Although one could argue that this problem has an "everyday mathematics" basis, it is not a problem that we are likely to encounter in our everyday life. It does build on the familiarity that children at this age are likely to have with respect to money, but it is an "artificial" problem. In everyday life, one either knows how much was spent or can find out by looking at the change given. I chose this problem because I think it opens up the way to some mathematics that we would expect in the context of mathematicians' mathematics--search for patterns, what if...?, what if not...?, generalization. It is a problem that does not conform to what many students have grown used to in a school mathematics class. For one thing, there is more than one answer possible. Yet, this problem was presented in the context of school mathematics. Many students were uncomfortable with this problem because it was not clear what was expected from them. Many of them never came close to any kind of "enjoyment" in looking for patterns and in sharing their findings with their peers. Instead, they wrote down two or three combinations and considered it done. They brought to this task the behavior that characterizes much of the culture of school mathematics (Davis, 1989). This paper explores some of the tensions between these apparently competing views of mathematics.

Three (at least) Kinds of Mathematics in Play?

School Mathematics

At a time in which many schools are to a more or less extent trying to adopt (or adapt) some of the messages in the reform documents (NCTM, 1989; 1991; NRC, 1989), talking about traditional school mathematics may begin to be inappropriate. This traditional school mathematics is often characterized by an overreliance on paper and pencil, meaningless computations, clearly formulated "problems," following prescribed algorithms, focus on symbolic manipulation deprived of meaning. Individual, seat work is emphasized. The teacher, the textbook, the answer key are the sources of authority to determine the validity of an answer (see Davis, 1986, 1989; Lampert, 1988; Schoenfeld, 1991, for characteristics of traditional school mathematics).

As I said earlier, many of these characteristics may not be so much the case, or rather these may still be present but competing with other features as teachers try to bring in change to their teaching of mathematics. Nowadays, if we walk into a mathematics classroom, we are likely to

see students working in groups, using manipulative materials, talking and writing about mathematics. They are often working on mathematically rich activities and the textbook (if used at all) is one more resource in the room. But, as a current project I am involved in shows day after day, the differences among classrooms that appear to all be engaged in this reform movement are abysmal. Classrooms are part of the larger context of a school: the school culture has definitely an effect in what these teachers do. Parents, administrators, and students form part of the school culture (well engrained by fifth grade) and are three clear forces in play. The classroom on which this paper is based is no exception. Hence, certain characteristics of traditional school mathematics were still in place. That the students were willing to play along with us as we tried to engage them in a different approach to mathematics learning and teaching does not mean that they saw what we were doing as "real" mathematics. Even the teacher was caught between our agreed attempt to develop an inquiry based curriculum and a more traditional one in which, for example, her students were supposed to learn the algorithm for long division since they had not learned it in fourth grade. Students who had otherwise evidenced an outstanding number sense were all of a sudden at a loss trying to follow the steps of a rather obscure and decontextualized algorithm.

My point is that even "reform" classrooms are embedded in a larger context at the basis of which lies the issue of what is mathematics. Many school districts are still using standardized tests to base many of their decisions. Teachers are caught in this dilemma. The issue of "back to basics" is not over. The values and goals of traditional school mathematics are still very much present. There is a wide spectrum of possibilities between traditional school mathematics and what seem to be examples of inquiry mathematics (Cobb, 1991; Cobb, Wood, Yackel, & McNeal, 1993; Richards, 1991).

Mathematicians' Mathematics in the School Context

If we were to have children in school doing mathematics as mathematicians do, what are some of the features that we would expect to see? Mathematics as a discipline deals with ill-defined problems; it requires time, persistence, and flexibility; mathematicians often refer to a certain element of "playfulness" in their work, of "messing around" with ideas in their search of justifications, counterexamples, and so on. Schoenfeld (1987) in describing how a group of mathematicians talk about their discipline, points out that these mathematicians speak frequently of the importance of collaboration in their work. The quotes selected by Schoenfeld show the excitement that these mathematicians share in their work and their feeling of belonging to a community with its own goals and values. Schoenfeld (1987, 1991) illustrates how some of these characteristics of mathematics as a discipline can be brought to the classroom creating in this manner a "microcosm of mathematical culture" (Schoenfeld, 1987, p. 213). Cobb (1991), Davis (1989), Lampert (1986;1988) also present examples of school mathematics that emphasize

joint meaning construction and in which children's ideas are not just acknowledged but actually used for further discussion. Cobb refers to the kind of mathematics he envisions as an example of what Richards (1991) calls inquiry mathematics:

Mathematics as it is used by mathematically literate adults. ... The language of mathematical literacy includes participating in a mathematical discussion, and acting mathematically - asking mathematical questions; solving mathematical problems that are new to you; proposing conjectures; listening to mathematical arguments (Richards, 1991, p. 15).

In summary, what would be some key characteristics of a classroom mathematics community in which children do mathematics as mathematicians?

1. Students and teacher engage in mathematical discussions.
2. Communication and negotiation of meanings are key features of the mathematical activity.
3. Students work in small groups and are encouraged to use and demonstrate to others their informal knowledge of mathematics.
4. Mathematical activities are academically challenging to encourage students to develop and share their own strategies to approach the tasks.
5. Students work on open ended problems and investigations.
6. Students are responsible for decisions concerning validity and justification.
7. Persistence is encouraged.

But, in mathematicians' mathematics, the participants have to a great (if not total) extent chosen to be there. They have certain general common ways of acting when doing mathematics that they have agreed upon (or that they accept as members of the community of mathematicians). School mathematics also has certain common ways, that children must (willingly or not) abide by. Children in a school mathematics class are not there by choice. The problems they are asked to work on, whether traditional or more reform oriented, are often chosen by the teacher. Furthermore, by fifth grade, most children have a well developed idea of the "proper" way to do mathematics in school. In our work, we seem to want to create within a classroom "some small piece of the real culture of mathematics, perhaps even an 'artificial' piece, but one that is at least true to the spirit of those who do use, or even create, mathematics" (Davis, 1989, p. 159). Is this possible?

Everyday Mathematics

By everyday mathematics I am referring to the uses that we make of mathematics in our everyday activities. A wide variety of studies have documented how people tend to perform virtually error-free mathematics in situations that they view as relevant to themselves and that pertain to their everyday activity (Carraher, Carraher, & Schliemann, 1985; Lave, 1988; Masingila, 1994; Nunes, Schliemann, & Carraher, 1993; Saxe, 1988; Schliemann, 1984).

In street mathematics, problem-solving activities are carried out in situations that are part of everyday life.(...) successful learning and problem solving in every day life may be explained by the preservation of meaning during problem-solving activities (Nunes, Schliemann, & Carraher, 1993, p. 142).

Yet, some of these studies have also documented that these same people had a lower performance on pencil and paper tasks that were designed to be "similar" to their everyday situations. An important aspect of everyday mathematics seems to be the fact that the subjects are often in control of the situation (Lave, 1988): they can choose to drop a certain problem solving strategy if they want to. They are not subject to a prescribed method they need to follow (as is often the case in school mathematics). They are free to invent their own methods of solution and these often reflect a great level of flexibility. Not only are they in control of the method of solution but often of the task itself, which they can modify or even abandon. In school, students have often very little control over their choice of problems and methods of solution.

On the other hand, everyday mathematics is often context bound, and as such may be limited in its generalization (Resnick, 1987). Also, in everyday activities, the mathematics is often hidden. We may not be aware that we are using mathematics, and in fact when pointed out, we may reject that what we were doing is mathematics. Can we say that we are using mathematics if we are not aware that we are? In the NCTM standards (1989), one reads "to some extent, everybody is a mathematician and does mathematics consciously. To buy at the market, to measure a strip of wallpaper, or to decorate a ceramic pot with a regular pattern is doing mathematics" (p.6). Is it?

Noss and Hoyles (1992) reject the notion that people who engage in activities in which there may be some frozen (Gerdes, 1986) mathematics present, are to a certain extent doing mathematics:

In our view, mathematics exists in the head, not in the street (markets). There is a wide range of activities that can serve as starting points for mathematical teaching, but that is not the same as arguing that the mathematics is in some sense "already there" waiting to be unpacked (Noss & Hoyles, 1992, p. 448).

Can we develop an approach to mathematics in school that builds on everyday situations and activities? As Hoyles (1991) asks, "*Is it possible to capture the power and motivation of informal non-school learning environments for use as a basis for school mathematics?*" [italics in original] (p. 149) There are a variety of proposals and programs that in one way or another (mostly depending on the developers' values and beliefs about mathematics) build on the contextualized aspects of everyday activity (see for example, Heckman & Weissglass, 1994; Mellin-Olsen, 1987; the Dutch project on Realistic Mathematics Education (Gravemeijer, van den Heuvel, & Streefland, 1990).

How far can we push everyday mathematics? Once we start mathematizing these everyday situations we may be losing what made them appealing in the first place, but we are gaining

(hopefully) in advancing the students' learning of generalization and abstraction in mathematics. In our work, what we are trying to do is to take some of these everyday activities as starting points and explore their mathematical potential from a mathematician's point of view, while staying within the constraints of school mathematics. Is this possible? This is the question that this paper addresses. Geometry allowed us to tap onto these students' knowledge about this topic based on their experiences in school and in their everyday life. We were able to combine quite successfully this knowledge with elements of a mathematics community. It also allowed us to partially tear down the wall that by fifth grade most students have build isolating what counts as school mathematics. The rest of this paper describes aspects of this work.

The Setting

This fifth grade class is in a bilingual (English/Spanish) school (K-8). There were 29 students (14 boys and 15 girls)--19 of Mexican origin, or other Hispanic origin; 5 Anglo-American; 4 African-American, and 1 Native American. Five of the students were predominantly Spanish speaking, and not all the others were bilingual. Most students in this class were from predominantly working-class families. The classroom teacher, a research assistant, and myself had collaborated the year before in that same school with the fifth grade class of that year. In that work we tried out some of our ideas towards the development of a learning community, in circumstances less than favorable (see Civil, Andrade, & Rendón, 1994). This experience was valuable as it allowed us to get to know each other better and to develop a plan of action for the coming year. This paper focuses on a section of this second fifth grade class, namely on some of our work in geometry (November 2 through mid December). We had been in this class since the beginning of the school year (see Civil, 1994), and thus we had been working on redefining what it means to do mathematics and on establishing social norms of behavior in a discussion in mathematics.

Our Work in Geometry

Our exploration of geometry took us in various directions; symmetry, angles, tiling patterns, tessellations, measurement, area and perimeter, scale drawing. After Christmas break we continued the work in geometry by introducing the students to logo. For this paper I will leave out the work on measurement and on logo and focus on what we did on patterns, tessellations and angles. What comes next is based on my field notes and those taken by the research assistant, copies of the students' work, and a task based interview of a group of four on finding angles on the pattern blocks.

Getting Started

I wrote down the term "geometry" on a transparency and asked them to, in their groups, write down anything that came to their mind in relation to that term. This technique of asking students

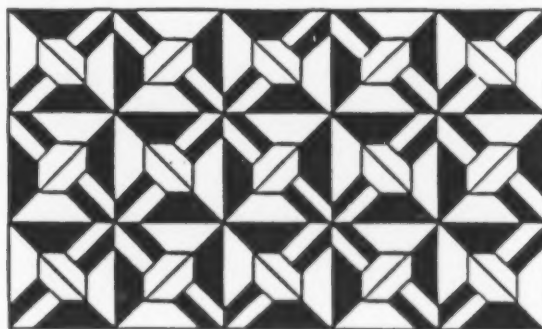
what they know about something allowed us to learn about some of their images about geometry. Do they view it as part of school mathematics? If so, what kinds of things come to their mind that may be the result of their prior exposure to geometry in school? Do they think of uses of geometry in everyday life?

Most of their entries listed vocabulary terms characteristic of what geometry in a mathematics class looks like: circles, shapes, length, angle. Some of the entries showed an awareness of geometry at places other than their school experience: in flying airplanes, in using computers for drafting (based on a commercial on local TV that shows people using computers to make house plans and design in general). One of the groups needed help since they said that they did not know what the word "geometry" meant. Another group wrote "The word sounds similar to Geography; she is changing the subject to geography instead of math. It has the three first words [sic] and the last one." Another group, after listing typical terms from school geometry, wrote: "making semi-creative pictures with annoying little wood blocks" and "listening to teachers tell you that geometry is everywhere." Another group also made reference to this "you use it everywhere." How much of this is it that they believe that one uses geometry everywhere, and how much is it that they are saying it because they think that this is what we want to hear? The student's observation of "listening to teachers tell you that geometry is everywhere" is very revealing, I think. There seems to be a tendency among teachers to try to convince students that mathematics is everywhere. Which mathematics is everywhere? Probably not the one that most students see at school.

Looking at Patterns

For the next few days we looked at a variety of patterns. Some of them were standard tessellations (regular, semi-regular, neither), but many of them were samples from Native American art (Navajo rugs, Hopi designs for shawls, Tohono O'dham basket designs, Pueblo pottery), as well as some Escher tessellations and some tiling patterns in buildings, sidewalks and in our environment in general. Much of this initial work was done as a whole group, with the students looking at samples of these patterns on the overhead projector. Our experience with whole class discussion until then had been somewhat "disappointing." Status in this classroom seem to be well established and it was always the same students who contributed to the discussion. We do not know whether it was the change of topic, or the fact that we had been working on appropriate behaviors, or some other reason that I am unaware of, but the fact is that as we started having students share what they saw on these transparencies, the rules of discourse opened up and all of a sudden students who had been quiet up to that point started to contribute. Looking for the basic pattern that keeps repeating occupied much of our work on these transparencies. For example, for the figure shown below, students came up with a variety of possibilities (one square tile, 2, 4 half tiles). This figure led to a discussion of rotational (with

subsequent exploration of concept of turn) and mirror symmetry. After a while, the class seemed to have split in three: a group who thought that the basic tile was being rotated throughout; a group who thought that the pattern was obtained by mirror symmetry; and a third group who did not seem to be interested in the discussion. As some students were talking about things being symmetrical, one of these "uninterested" students softly asked his neighbor "what is symmetrical?", "it repeats itself, it mirrors itself", his neighbor replied.



(From O'Daffer, P., & Clemens, S. (1992). Geometry: An investigative approach (second edition). New York: Addison-Wesley) (p. 101).

For homework, the teacher asked them to look for three different patterns around their house, neighborhood, or school, draw them and describe them. Not only did most of the students had their homework by the deadline (something quite unusual in this class), but their write-ups showed a wide variety of places that these students had looked at in search of patterns (in their houses, the mall, the restaurant, clothing, garden, books, jewelry). Mathematics is often characterized as the science of patterns, and in many classrooms from Kindergarten on, a lot of time is spent looking at and for patterns. In reading these fifth graders' homework papers, I could not help wondering what it is that some of them thought a pattern was. For example, if the cover of a bed is $\frac{2}{3}$ red and $\frac{1}{3}$ purple, is this a pattern? or if the "walls of my room have many colors," is this a pattern?

I am afraid that often we leave things at the surface level, that is we find a pattern but we do not go beyond this, and ask, but why is this pattern occurring. It is the analysis that takes place once a (potential) pattern has been observed that I consider as doing mathematics. Unfortunately, I think that in our excitement in seeing students engaged, we often leave out the processing time. In fact the teacher in this classroom has repeatedly remarked, as she reflects on

the implications of reform in mathematics education, that she lacks time for processing with her students. She feels that she needs to move on and that her students get restless during discussion time.

Working with Pattern Blocks

The teacher and I wanted to work on the concept of angle and on which regular polygons tessellate and why. We saw pattern blocks as a good manipulative to help introduce some of these concepts and reinforce work with fractions. Each group was given a triangle and trapezoid and was to say how these were the same, and how they were different. We did this as a whole group with different students contributing their ideas. This allowed me to discard color and thickness as non mathematical properties for this task. That is, we mentioned them once, and that was it. Fractions were introduced by my writing $1 \text{ Trapezoid} = 3 \text{ triangles}$, thus, $1 \text{ triangle} = \frac{1}{3} \text{ Trapezoid}$. Some discussion took place at this point, as some students suggested $\frac{1}{2}$, others $\frac{1}{3}$, and others $\frac{1}{4}$. Let's pause here for a moment. Shouldn't we expect that most students by fifth grade can see that if 1 trapezoid equals 3 triangles, then 1 triangle equals $\frac{1}{3}$ of a trapezoid? Maybe this task is quite artificial, maybe one could argue whether it is of any relevance to know how much of a trapezoid is one triangle. Maybe those same students who thought that it was $\frac{1}{2}$ or $\frac{1}{4}$ would be very able to deal with a similar situation in a context meaningful to them. In fact, letting students explain why they thought that it was $\frac{1}{2}$, $\frac{1}{3}$ or $\frac{1}{4}$ is important because it allowed us to find out how these students were interpreting the task. Unfortunately, it is not clear to me what the reasoning was for $\frac{1}{2}$; for $\frac{1}{4}$ it seems to have had something to do with arranging triangles around the trapezoid (it takes five of them, not four).

Both the teacher and I believe that these students should be able to reason that $1 \text{ triangle} = \frac{1}{3} \text{ trapezoid}$ (once the context has been clarified). In fact, part of our rationale for using pattern blocks was to revisit these students' understanding of fractions. We often went back and forth between what could be considered more traditional content that we thought students should know (such as a flexible understanding of fractions).

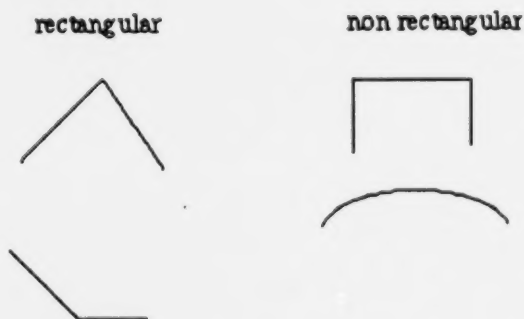
After this initial task, students were to work in their groups on comparing the hexagon and the triangle and then the blue parallelogram and the triangle. The task seems simple on the surface and yet it is open enough to allow for students to take it in different directions. Thus, one group became involved in one of our first instances of what I would describe as a mathematical discussion as one of them was saying that the hexagon is 6" around and the other two members in the group were saying that it was more like 1.5" around. How had inches got into this? This group had noticed that another group had decided to take a ruler to measure different parts of the pattern blocks. This group took this idea and while one of the students (J.) was measuring around (the perimeter), the other two were measuring across (along one of the

diagonals), yet they were both using the term around and that led to the discussion. In their final write-up, they wrote:

We have measured the hexagon with the triangles. It takes six triangles to make a hexagon. It is one and a half inches around the hexagon.

The student who was measuring the perimeter and correctly saying that it was 6" around happened to be a student who often got in trouble mostly by making up stories. One of J's most frequently used stories was to say that the principal was related to him (which he was not). J. would often interject gory or vulgar comments during class discussion. He was not very well accepted by his peers, who avoided him whenever possible. On the other hand, one of the students who was saying that it was 1.5" around, was very well liked by everybody and had a very high social and academic status in the classroom (in fact both students who said it was 1.5" around were in GATE (Gifted and Talented Education)). I took part in this group's discussion and I tried bringing the other two students to pay attention to J's idea. But their final write-up reflects the interpretation of the two students with "higher" status.

The groups' write-ups give us an insight in their use of vocabulary to describe these shapes and their properties: "the hexagon in the middle is $1 \frac{7}{8}$ of an inch." "los verdes son mas chiquitos y cuando los pones juntos parecen pedacitos de naranja" [the green are smaller and when you put them together they look like pieces of an orange] "the hexagon has double the amount of sides than the triangle." "Both are three dimensional." Talking to students about their findings also shed some light into their ideas about geometric concepts. One boy referred to the triangle and the blue parallelogram as being "rectangular." What did he mean by this? He elaborated and shared the following:



Patterns Revisited: Tessellations

After a few days exploring the concept of area through a variety of tasks (carpeting the room, geoboard), we went back to the transparencies of tessellations shown at the beginning of the module. I started the discussion on tessellations by showing them again some of Escher's work as well as samples of tessellations created by students. Then, on the overhead, I put a

transparency showing a tessellation with triangles and squares (see end of paper for copies of these, de Cordova, 1983) and asked them to find the basic repeating tile. In my field notes, I wrote:

What made this discussion very interesting was the degree of participation: students who had never been at the overhead all of a sudden wanted to come and share their thinking. I had Elena² (who came up with almost the basic tile), then Claudia (who found the smallest tile on the first pattern); also Marisela, Peter, Marcos (who came to the overhead and spent a "long" time trying to reconstruct what he had thought of; students started getting antsy and he finally gave up)... and the usual crowd. (Field notes; Dec 7)

But who were these students? A key aspect of our work is to get to know as much as we can about each individual student. At the beginning of the year, the research assistant interviewed each student using a protocol adapted from the one used in the larger project (Funds of Knowledge). Our goal was to gain some information about the students' funds of knowledge in order to build on these in our design of mathematics tasks. Of the five students mentioned earlier, who came to the overhead to share their thinking, three of them (Claudia, Elena, and Peter) had never shared anything in the mathematics class. Peter disliked mathematics very much and insisted that he was not good at it; Claudia was a very good softball player with very little confidence in her academic skills; Elena was very quiet. Of the other two (Marisela and Marcos), Marisela was predominantly Spanish speaking and quite open if asked directly (but not in front of the whole class); Marcos had been labeled learning disabled.

I used the tessellation with triangles and squares to introduce Schafli's symbol (de Cordova, 1983), that is, in this case: (3, 3, 4) (triangle, triangle, square). The students then worked in their groups with other examples of tessellations looking for the basic repeating tile and determining Schafli's notation. On one of these tessellations there were two types of vertices (see end of paper, T3). The first person to come to the overhead circled the hexagon as the repeating tile. One of the students then said "that doesn't work; it doesn't repeat." At this point I acknowledged that this was the kind of behavior that we were working towards. Not only did this student commented on what someone else had done but offered a reason for why he disagreed. As I wrote in my field notes reflecting on this incident:

They have to see and listen to what their peers have to say instead of being so self-centered that all they want is to get their word in.

An important aspect of our vision of a mathematics learning community is to change the rules of discourse. Traditionally, students in a mathematics class are not used to listening to each other's ideas. What I find particularly frustrating is that quite often when one student is sharing his/her thinking outloud, many (if not most) of the other students have a tendency to "switch off" as if the conversation had nothing to do with them. During many of our class discussions,

² All students' names are pseudonyms.

students would contribute to the discussion, but their contributions were often to get their voice in, but hardly ever to build on, agree or disagree with what someone else had just said. This pattern of discussion is likely to be different from what mathematicians engage in as they are working on a problem. For one thing, most of these students were not used to the idea of "an argument" in mathematics. In fact, many of them were constantly amused by my use of the term "argument." But an even more important issue may be the fact that mathematicians (or even adult non-mathematicians) and ten-year-olds have different goals and different reasons for wanting to be heard in a mathematical discussion. For these students, the actual mathematical question may not be so important. What counts is their status in the classroom, who gets to speak, and how to react to what was said (not as a function of what was said, but as a function of who said it). In addition (and this would be true of adults too), the issue of being right or wrong is important, no matter what we say to try to change this perception. It is not easy for anyone to advance an idea and see it fall apart a few seconds later. Finally, this kind of discussions takes time. Many students did not consider the topic under question so relevant to them as to deserve spending time listening to their peers' comments and then building on those. This is not to say that these students had no persistence or could not engage in logical arguments. We witnessed several instances of this, and more so as students worked on their logo projects. I believe that if we observed these same children in their everyday games and activities outside school, when the task they are engaged in is the center of their attention, we would see a behavior more similar to the one that mathematicians bring to their discussions with colleagues.

Instances of Students "Getting Into It"

In reading accounts of mathematics as a discipline (see, for example, interviews to mathematicians in Albers, Alexanderson, 1985 and Albers, Alexanderson, Reid, 1990) and in talking to mathematicians, the excitement of "getting into a problem" characterizes their approach to their work; curiosity is a driving force for them.

In our attempt to develop a microcosm of mathematical practice, we look for instances of students "getting into a problem," making discoveries, pursuing an idea, enjoying themselves. Here I present very brief examples of such happenings; these have to be seen as part of the larger context of changing the norms of what it means to do mathematics.

1) In looking for the Schafli symbol for a tessellation with two kinds of vertices, students came up with: (3, 3, 4, 3, 4) and (6, 4, 3, 4). Right away a student observed that "they add up to the same." Sure enough, $3 + 3 + 4 + 3 + 4 = 6 + 4 + 3 + 4$. I told them that I had not noticed that. Another student then said "you don't know? you're the teacher!" "No, I did not know, and this is why I liked it for them to share their ideas, their discoveries, so that we could all learn from each other," I told them.

2) As I asked them if they could think of a shape that would not tessellate, many of them right away suggested a circle. For homework I asked to look for a polygon that did not tessellate. The next day two students who had worked together on this, shared their pentagram (five pointed star) as an example. There was some discussion as to whether this was a polygon or not. One of the two students said that her brother who was in seventh grade had helped them and that he had said that it was a polygon, but she said "in any case, I can draw a five pointed star, no crossing [since this was the point of disagreement for whether it was a polygon], and it won't tessellate," and proceeded to do that on the board. As the students looked at the original five pointed star, several of them noticed that it was like the symbol used on the Chrysler cars; some students remarked that there is a pentagon inside; some looked at the shape as being made out of triangles. Students volunteered these contributions. They appeared to be genuinely interested in the shape that their classmate had drawn. There was some discussion as to the meaning of the word "pentagram" as the teacher remarked that she had always associated it with music (the two students who shared the five pointed star referred to it as a pentagram). One of the students all of sudden came to the board and drew a six pointed star by drawing two intersecting triangles. Right away students noticed that in this case there is a hexagon inscribed. One of the students who had shared the pentagram said that maybe this one was called a hexagram because it had six points and since the other one was called a pentagram. She also added that "gram" probably meant "point." Talking about the possible meaning of terms in mathematics and encouraging students to come up with their own terms help nurture the idea that mathematics is a cultural artifact and that as such they can also be a part of it.

What I think needs to be emphasized in this brief episode is that there was almost full class participation on a task grounded on two students' contribution. Furthermore this participation took place in a "healthy" way. There were no instances of put-downs and the whole participation revolved around ideas discussed in mathematics. In order to develop a mathematics classroom community, the class has to develop a feeling of community, something that we have found rather hard to do in this school (at the fifth grade level, which has been our only experience).

The pentagon inscribed in the five pointed star led to our next task: the students proceeded to explore whether regular pentagons would tessellate. The students soon realized that they did not seem to tessellate as easily as the regular hexagons (they had worked with these as part of their exploration with pattern blocks). They were quite resourceful in trying to "force them" to tessellate: overlapping them, arranging them in a circle and not going inside the circle, using the tan parallelogram (from the pattern blocks) to fill in the gap. Students were eager to share their findings, their attempts to make the pentagon tessellate. They also became involved trying to tessellate with a T-shape; students soon made the connection to the computer game Tetris and found different ways to tessellate with this shape. After a while, we reached an agreement that

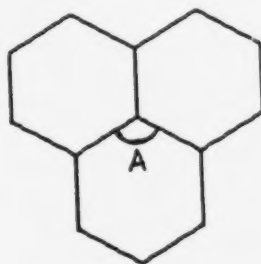
regular pentagons would not tessellate according to our rules of what a tessellation is. I then asked them "how come the regular pentagon does not tessellate while the regular hexagon does?" Some students said that it had something to do with the corners. Indeed it does. Then, one of the students (F.) said that it was because of the odd number of sides: the pentagon has 5 sides while the hexagon has 6. This is a nice example of a conjecture that, in my rush to get to the concept of angle, I ruined. What I should have done is to turn F's conjecture to the class and have them explore it. Probably they would have come up with a counterexample. What I did, instead, was to provide myself the counterexample by arranging on the overhead a tessellation made of triangles (odd number of sides). F. right away said "oops, there goes my theory."

Angles on the Pattern Blocks

What I had in mind was to have them find out the angles in the different pattern blocks and then come back to the regular pentagon and find its interior angle. Then we would go back to tessellations and see what happens around a given vertex. The angle around a vertex is 360° , hence the angles of the regular polygons that do tessellate have to be factors of 360. The teacher told me that they had not done much on angles. I soon found out that they seemed to have at least two pieces of knowledge: that a corner on a square is called a right angle and/or a 90° angle, and that a whole turn is 360° (by putting for example four squares together and looking at the angle around the vertex in common). Well, let's use this information to find angles on the other pattern blocks:

a) I put 6 triangles together (forming a hexagon), point at the angle around the center: " 360° ," they say. So, "what about each of these angles on the triangle?" Some students said something about all angles being 180° ; a student says 60, "why?" because three 60's is 180. I point out that another way of finding this result is $360 \div 6$, since there are 6 triangles at the center.

b) I then move to the hexagon and put three hexagons and point at the angle A (see figure below). What I was hoping for was: $360 \div 3$, hence the angle is 120° .



But instead, students came up with several other things related to angles on the design shown, but not necessarily to A directly. To make it easier to read, I will reconstruct the dialogue from my field notes:

N: 6 times 180 (1080) will give us the angles inside the hexagon (because of the six triangles that I had previously arranged as a hexagon); then multiply this by 3 because there are 3 hexagons.

Me: What will the 1080×3 tell us?

This was a genuine question on my part; I was not sure where he was going with his line of reasoning and I wanted to pursue it. But N. became all flustered (he often reacted like this when questioned about anything) and said:

N: I don't know, it was just an idea.

Despite my efforts to convince him that I was genuinely interested in his idea and that I wanted him to carry it further so that I could understand it better, we did not get any further on this.

M: I think that the angles inside are 360 because of the 6 triangles and at the center point being 360.

R: I think that it will be 720 for the angles in the hexagon (the six "A") because for every triangle, two of the three angles touched on the border of the hexagon. So, since at the center it was 360 coming up from 1 angle from each triangle, at the border it would be twice as much, thus 720.

At this point in the discussion, very few people were following it. In fact the only students who were participating were all GATE students; whether justified or not, accurate or not, the fact is that students have a very good idea of where each one stands, academically and socially in the class. During our work on finding the basic repeating tile on tessellations we had succeeded in attracting the participation of students who hardly ever got their voice heard in mathematics. Now, in the task of finding the angle A, only a few students seemed to be with us. Was it because this task looked more "mathematical" to them, and hence they felt they could not contribute to it? I guess my question is "how can we ground the discussion in such a way that students are engaged in hard mathematical ideas yet they all feel invited to participate?" The students who did contribute seemed to have certain rules of mathematical discourse down; they knew how to reason, even though their reasoning may be faulty. They appeared to be comfortable talking mathematics.

Since I was aware that we had lost the majority of the class, I decided to turn the task to their small groups and have them write down their findings. The task remained the same: find the measure of the angle A on the hexagon (see Appendix for sample write-ups).

Working in groups is not easy. Developing appropriate group behavior is not an easy task. For this particular period of the year, the teacher had made the group assignments (in consultation with the research assistant and myself). I think that the topic of small groups is perhaps the one that most markedly differentiates what takes place in the classroom from what could be taking place outside school or in mathematicians mathematics. The latter have a common goal that may take precedence over their personal differences, the goal of solving a problem. Furthermore, they have certain flexibility in their choice of who they want to collaborate with. As children, outside school, they are likely to have a choice as to which activities to engage in and with whom. As adults, in the work place, they may not have so much flexibility either, but again, the pursuit of a common goal (the completion of the task in a successful manner) may take precedence over their differences. But in the classroom, the completion of the task does not seem to have the same leverage as in the outside world. Children are often in groups who are not of their choice and working on tasks which they did not choose. We (adults in the classroom) may have academic goals in mind. But the reality, at least in our experience in this school, is that the children's goals take precedence and that these goals are often a function of who the students in the group are and what their status in the classroom is.

So, going back to the task on finding the angle, I decided to break the rules of grouping. R and M who had both contributed to the discussion and who were genuinely interested in the task were also very good friends since they were very young (they are neighbors). They were usually in different groups and on this occasion R. was in a group in which no one else had any interest in working on this task. R. was evidently frustrated and requested my permission to join the group where M was, which I granted. Well, was F. upset! F. was one of the "high achievers" too and when he saw a group with R and M he went immediately to the teacher to complain. He said "it is not fair to have this group because M. and R. are the two smartest girls, people, in the class." As I wrote in my field notes:

I may have committed a faux pas, but I was getting tired of the lack of cooperation in some groups. R. is totally isolated in her group with K. being a real problem most of the time.

The issues of competition and copying among groups were constantly present. Are most ten year olds competitive? Is it because in this school the sports program is very important and competition is an issue there? Or is it because, as F. explained to me a few weeks later on a related incident, they were concerned about the issue of grades?

A Task Based Interview on Finding Angles on the Pattern Blocks

What are students learning about mathematics in this class? How can I tell? One clear concern of mine since I started working on this project is how to document what students are learning (or not) in mathematics. Yes, we see students participating, engaging in activities, talking mathematics (sometimes), and yet, what is it that they are learning? I thought that interviewing a group of four children may give me a chance to look at what was going on in a more controlled environment. I interviewed this group twice, one on tasks around the concept of area and perimeter, and the second interview on finding the angles on the other pattern blocks besides the square, triangle and hexagon that we had done in class. There are lots of logistical issues surrounding the interviewing process, but here I just want to briefly summarize some of my findings in terms of what these children did. In fact I will mostly focus on one of the four children, Andrew. The four students had been in the same group for some time. They got along quite well, except for some sporadic animosity between one of the boys and one of the girls. The four children were very nice, sweet, and participated regularly in class discussions. Andrew was quite an interesting boy in that he was usually in a good mood but quite unpredictable in how he chose to participate in class. At one point the teacher had told me that it was because he was very intelligent and bored easily. At that time I was skeptical, but as the year went on, I realized that Andrew was very insightful, and had an amazing number sense. He did not like writing at all. He was argumentative, and as such, he could easily get into a mathematical argument. The most poignant incident with him was one day when the seven students who were in GATE were pulled out for GATE, he said to the teacher "how come I don't get to be in GATE so that I can get smart?" I felt for this child, and for all the others in this class, as I once again saw how unfair these pull-out programs can be, and the effects they have on the children.

Finding the angles on the blue parallelogram turned out to be quite easy. They did that by using the triangle as their measuring tool. The difficulties started with the trapezoid. They covered it with three triangles and kept referring to the point where the three meet and saying that it was 180 (but this is not one of the angles of the trapezoid); Jennifer said that since it took 3 triangles, "it was 180." They seemed to be looking for the total sum of the angles (which for the blue parallelogram, they had established it was 360). Then Andrew said that it was $530 - 360$ from the blue parallelogram plus 180 from the triangle (since these two shapes do cover the trapezoid). After I asked them how many corners there were on the trapezoid, they focused on looking for the measurement of these corners. The 60° ones were easy for them. The problem was with the 120° . They tried to fit a triangle, then Andrew said that it was 90° , to which the others disagreed; Jennifer said " 90° is the right corner"; then Albert mumbled 120; all of sudden their level of

excitement went up as they realized that the blue parallelogram fit exactly that corner and they knew from before that that angle was 120° .

There was quite a lot of chaos all throughout and not everybody was following what was going on. In fact, most of the talk was made by Andrew. The students were not too clear on how using the triangle as a measuring tool for the angles could be helpful; they knew a few facts, such as 180° in a triangle, and used this sometimes; at other times, they focused on covering the pattern block completely and then adding total angles to get the total sum, and yet at others they focused on the length. An example of the latter occurred as they were working on looking for the angles of the tan parallelogram. Andrew right away said that "one side is 30 and this side is 60," and his logic for this is that one was skinnier than the other. After a while, Jennifer said that it was four 60's, or 240. Why four 60's? She explained that she had matched each side of the parallelogram with one side of the triangle. Since they are the same size, she concluded that each angle was 60: "but it [the triangle] doesn't fit inside, so, I have no idea" (she added).

Confusion between angles, sides, and what exactly it means to measure an angle was taking place. Since Andrew was still thinking that one was 30 and the other was 60, the sum of these four angles would then be 180. (He had given a convincing argument for 30° for the small angle, and was confident about it.) This 180 seemed to be a magical number, because then the four of them reinterpreted the task as looking for two possible numbers for these angles with the condition that the total sum is 180. It took some trying out ideas till they decided that the bigger angle was larger than 90 and smaller than 180. Andrew then suggested 120; I brought this up to see if they could check this idea with the tools they had, but then Andrew said:

Andrew: No, I think it's 130

Jennifer: 130 is odd though, and everything else

Andrew: Ok, then it's 120

I am guessing that what Jennifer meant by 130 being odd is that it was a number that had not come up yet; so far the angles had been 60, 120, 90. Andrew was not just giving up to 120. He went on thinking about this, and all of a sudden as he added $120 + 120 + 30 + 30$ and came up with 300, he said "we need 60 more, so it's 150 and not 120." He sounded confident, as if everything had all of a sudden clicked. I was impressed by his persistence and his ability to carry out his arguments. Andrew who was usually the first one to join in the distraction and the "goofing off" as soon as someone started it, was very serious and involved in the mathematics during this session. Of course the interview left me with serious doubts as to what it is that these students (and the rest of the class) were getting out of this measuring of angles. But, it also allowed me to see and listen to four children as they really tried to make sense out of the mathematics in the task.

Conclusion

In looking back at what I saw happening in our work in geometry in this fifth grade class, the most salient image is the fact that the participation patterns opened up, allowing for more students to take part in the discussion. Through our work we had glimpses of a learning community in which students who wanted to convey an image of "tough" and uninterested in academic matters, dropped that role and became caught in the mathematical discussion; students who had not said much up to that point, opened up slightly either through direct participation or through helping someone else. Students noticed what their peers were doing and often shared or "borrowed" strategies and resources (Roth, in press). Questioning and conjecturing took place; "technical" vocabulary (e.g., polygon, regular, trapezoid, quadrilateral) was discussed by students and definitions were reached through these discussions.

But these participation patterns varied considerably with the kind of activity the students were engaged in. Is there something in the transparencies of patterns and tessellations that made it conducive for almost everybody to participate? Yet, as soon as we became more involved in the exploration of angles and why some shapes tessellate and other do not, many students withdrew from the conversation. Similarly with the work on the geoboard: students were eager to contribute shapes and even shapes of certain area, but as we moved to "proving" why they were of certain area, or finding all triangles of a given area, once again several students withdrew. As I said earlier in the paper, a key question for me is: how can we ground the discussion in such a way that students are engaged in mathematically rich ideas, yet they all feel invited to participate?

The nature of the mathematics involved certainly played a role in the patterns of participation. But several other key factors affected these patterns, and hence the development of a learning community. Unfortunately, for some of these factors we feel quite powerless in terms of changing the conditions in the course of a year:

- The culture of the school: through my involvement in a different project, I have been visiting fourth/fifth grade classrooms in several schools in our area. This has allowed me to see the large disparity among school cultures. The school in which the work reported here took place, is characterized by operating like a large family, but a family that is somewhat "unruly" (see, Civil, Andrade, & Rendón, 1994, for some description of the school atmosphere). Sports play a key role in the life of this school. Students active in certain sports (e.g., football, basketball) are likely to be automatically popular.

- The culture of the classroom: naturally, the culture of the school finds its way into the classroom. As we tried to change the norms of discourse in the mathematics classroom, we were faced with the students concept of status in the classroom hierarchy. Thus, for example, in mathematical discussions, students would often take for granted what GATE or "popular" students said, independently of its mathematical correctness.

- This approach to mathematics learning and teaching requires considerable risk taking on the part of the students. Not only did we have to tackle the issues of "copying," competition versus cooperation, but also we had to work on developing an atmosphere in which "being wrong" was to be seen as something valuable to all of us in our route towards learning.

- In a mathematics classroom community, students would spend time exploring problems. Task persistence and spending time certainly characterize the work of most mathematicians. This is in clear contrast with what traditionally takes place in schools: get things done fast, move on. Hence, although the proposed tasks may be mathematically rich, unless we can engage the students in spending time on them, exploring and reflecting, we may be missing the point.

And in fact, this "missing the point" brings me to my closing point: what mathematics are these students learning? How can I tell what (if any) understanding they gained? As long as we were looking at tessellations, making connections to similar patterns in our environment, creating their own, and so on, many students took part in the activities. Yet, I felt that I had to push these tasks in order to bring out [my view] of the mathematics in them. As Hoyles (1991) writes in relation to I.S.M (Informal Learning Environments for Mathematics within School):

The rationale for I.S.M is to provide vehicles for generalization and the formalization of these generalizations.

(...)

It is pedagogic intervention which imposes the mathematical structuring and provokes the pupils' awareness of the underlying mathematical ideas. (p. 149)

Similarly, Sierpinska (1995) writes:

To see mathematics in a situation, one must already know some mathematics and be "mathematically tuned". It is hard to imagine that in a situation which would be completely open, not labeled as intended to be mathematized, the activity and curiosity of the children would lead them to uttering mathematical statements and asking mathematical questions. (p. 4)

Although one goal in our work is to have students taking charge and becoming the ones who mathematize, I see my role as the facilitator of this role by modelling some possible mathematizing. In doing this the number of students who actively participated decreased. But, what did the different students in this classroom "gain" from this work? The task-based interview gave me a place to start addressing this question. A next step would be to focus on what kind of understanding specific students are developing by following these students from day one. A model for mathematical understanding such as that proposed by Pirie and Kieren (1989; 1994), in which understanding is seen as complex, growing, and made of different levels may prove helpful in trying to describe what sense students are making of these experiences.

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Appendix

Write ups on Finding the Angles on Pattern Blocks

1. The hexagon needs 6 triangles to fill it. Each triangle is 60° ; so if you put all six triangles on the hexagon then you would end up with 360° .

2. Pentagon. You can not build up a pentagon with triangles but I'm trying anyways. The pentagon takes 5 triangles but there is little spaces between each triangle so we figured out that all those spaces put together equal one more triangle. All together it still equals 360° .

We think that there are 720° in the whole hexagon. This means that there are 120° in each point. We came to this conclusion using many different strategies.

- #1: A trapezoid is 360° , two trapezoids ($360^\circ + 360^\circ = 720^\circ$) are one hexagon (720°).
- #2: Assuming that a hexagon is 720° we divided 720 by 6; the answer is 120° , we timesed 120 by 3, the answer was 360, we already knew that the vertex was 360° ; again this proves our point.
- #3: We multiplied 120° by 6 to see if your previous answer of 720° was correct. It was.

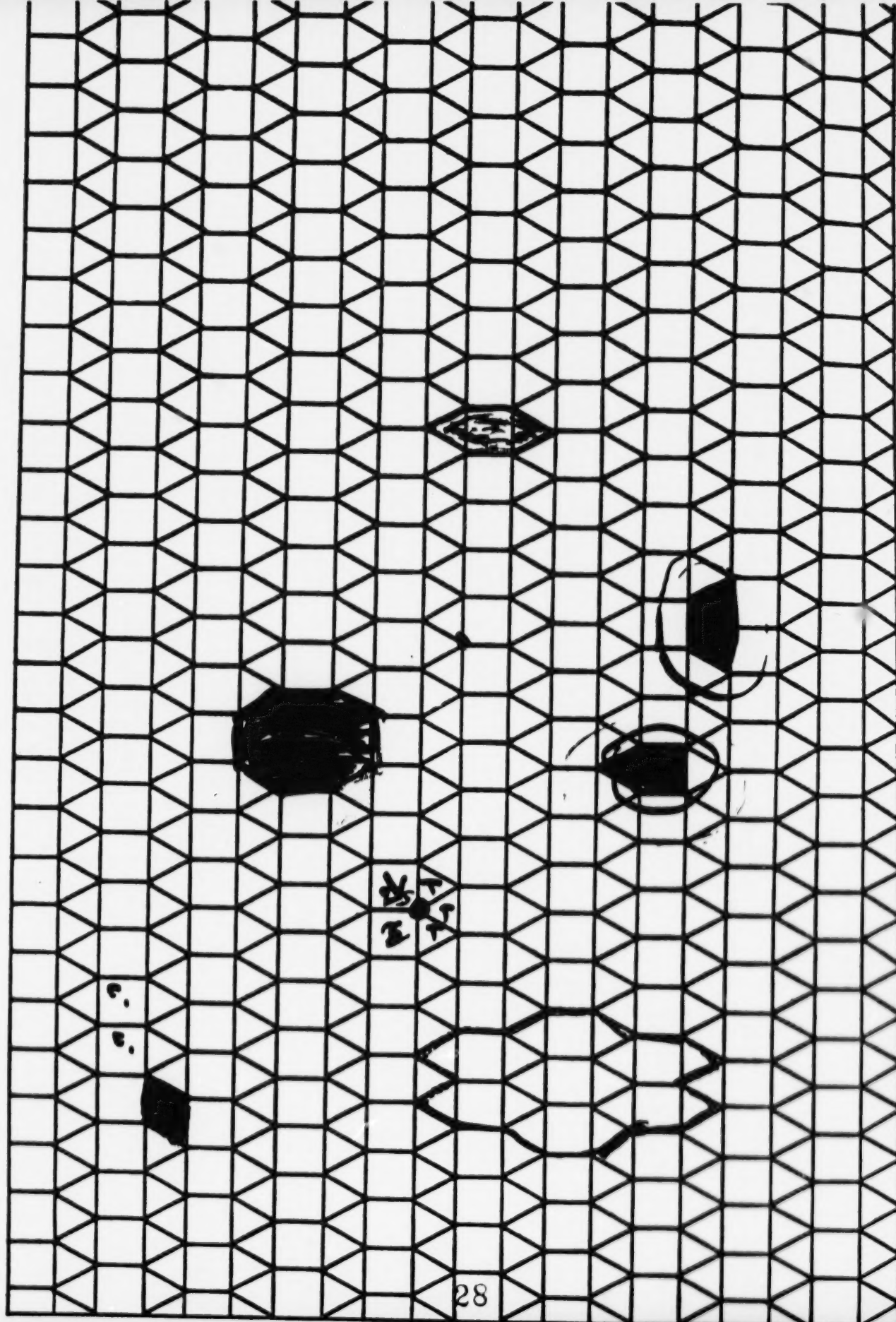
Nosotros pusimos los triangulos en el hexagon. Seis triangulos cubren un hexagono; dos triangulos caben en un angulo del hexagono y son 120. Tambien todos los angulos son 720. Lo hice por multiplicar $120 \times 6 = 720$.

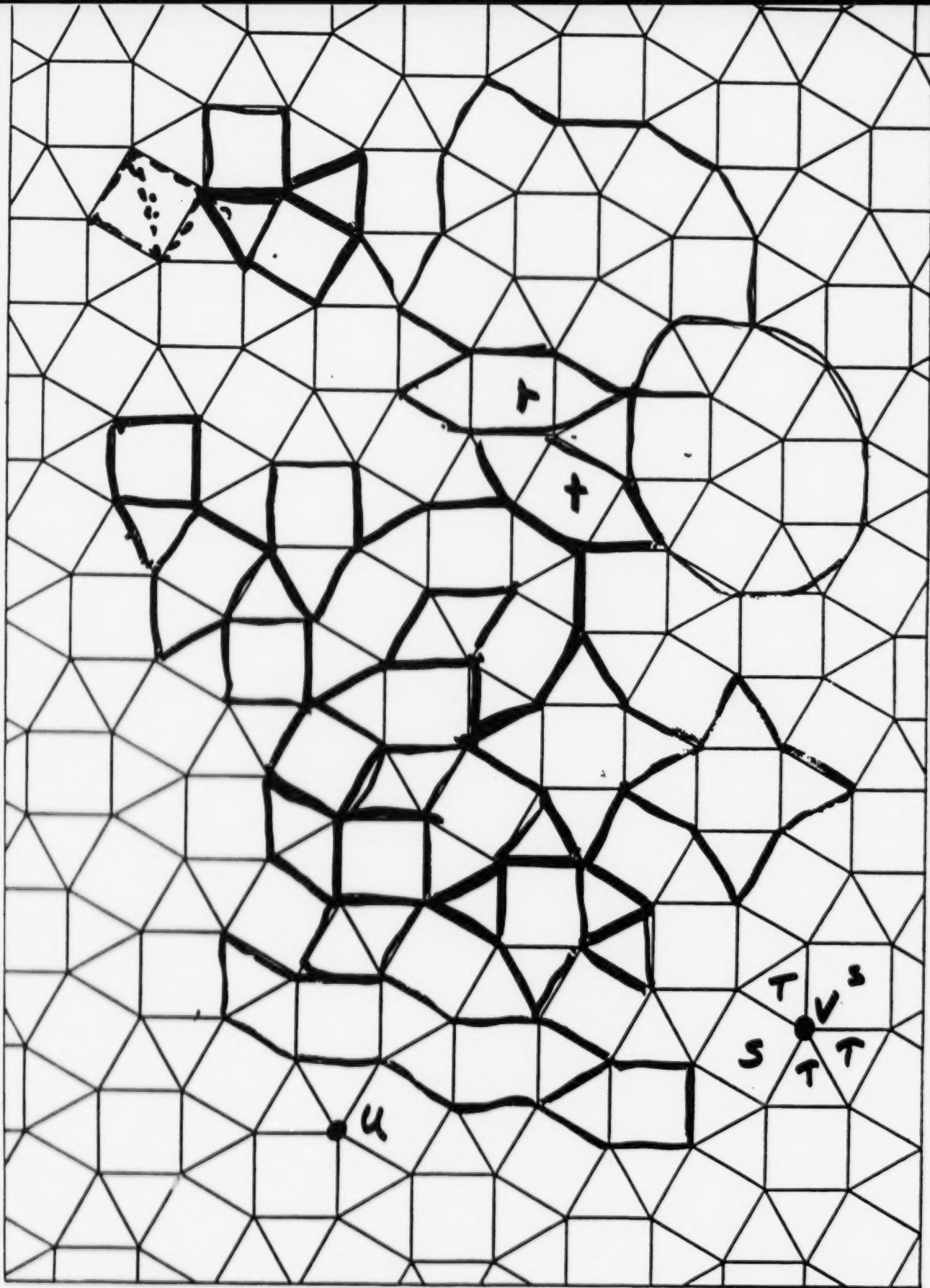
[We put the triangles on the hexagon. Six triangles cover one hexagon; two triangles fit in one angle of the hexagon and they are 120. Also all the angles are 720. I did it by multiplying $120 \times 6 = 720$]

The hexagon needs 6 triangles to fill it. We think it's 2,160. Each triangle is 60% so if you put all six triangles on the hexagon then you will get 360%

The angle of the hexagon is 120° . The angle of all of the six sides or should I say the six points equals 720° .

Because we put the square on the side of the angle, and it seems to be smaller than the angle of the hexagon. And it should be more than 90° .





~~12~~
T3

